التفاضل

المحاضرة الاولى

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Second Term

Calculus

Lecture 1

⇒ المنهج

- Hyberbolic Functions الزائدية , Inverse Functions. العكسية
- Integrals.
- Applications of Integrals.

⇒ Math : 150 marks → 100 marks Final

50 marks Term work "25 Algebra"

25 calculus

[1] Hyberbolic Functions

⇒ * $\frac{e^x - e^{-x}}{2} = \sinh x$ (hyber sin) , $\frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x} = \operatorname{csch} x$

* $\frac{e^x + e^{-x}}{2} = \cosh x$ (hyber cos) , $\frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x} = \operatorname{sech} x$

* $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$\Rightarrow$

$y = F(x)$	$y' = F'(x)$
$\sinh u(x)$	$\cosh u(x) \cdot u'(x)$
$\cosh u(x)$	$\sinh u(x) \cdot u'(x)$
$\tanh u(x)$	$\operatorname{sech}^2 u(x) \cdot u'(x)$
$\coth u(x)$	$-\operatorname{csch}^2 u(x) \cdot u'(x)$
$\operatorname{sech} u(x)$	$-\operatorname{sech} u(x) \cdot \tanh u(x) \cdot u'(x)$
$\operatorname{csch} u(x)$	$-\operatorname{csch} u(x) \cdot \coth u(x) \cdot u'(x)$

$\Rightarrow$ Find y'

(1) $y = x^3 + \sinh x^2$

$y' = 3x^2 + \cosh x^2 \cdot 2x$

(2) $y = \ln x \cdot \tanh x\,(3x+1)$

$y' = \frac{1}{x} \cdot \tanh(3x+1) + \ln x \cdot \operatorname{sech}^2(3x+1) \cdot 3$

(3) $y = \cosh^3 2x + \sin 2x \cdot \operatorname{sech} 3x$

$y' = 3(\cosh 2x)^2 \cdot \sinh 2x \cdot 2 +$

$\cos 2x \cdot 2 \cdot \operatorname{sech} 3x + \sin 2x \cdot -\operatorname{sech} 3x \cdot \tan 3x \cdot 3$

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2 Inverse Functions

$\Rightarrow$ $y = F(x) = 2 + \frac{1}{x}$

x	y
-1	1
1	3
2	$\frac{5}{2}$
3	$\frac{7}{3}$

$\Rightarrow$ $y - 2 = \frac{1}{x}$

$x = \frac{1}{y-2}$, $F^{-1}(x) = \frac{1}{x-2}$

$\Rightarrow$ If $F(x)$ is Function then it's inverse is denoted by $F^{-1}(x)$ and it can be obtained by obtaining x interms of y.

$\Rightarrow$ Trigonometric Functions , Inverse Trigonometric Functions

Trigonometric Functions	Inverse Trigonometric Functions
$\sin x$	$\sin^{-1} x = \text{arc} \sin x$
$\cos x$	$\cos^{-1} x$
$\tan x$	$\tan^{-1} x$
$\cot x$	$\cot^{-1} x$
$\sec x$	$\sec^{-1} x$
$\csc x$	$\csc^{-1} x$

⇒ Hyberbolic Functions , Inverse Hyberbolic Functions

$\sinh x$	$\sinh^{-1} x$
$\cosh x$	$\cosh^{-1} x$
$\tanh x$	$\tanh^{-1} x$
$\coth x$	$\coth^{-1} x$
$\operatorname{sech} x$	$\operatorname{sech}^{-1} x$
$\operatorname{csch} x$	$\operatorname{csch}^{-1} x$

$y = F(x)$	$y` = F`(x)$
$\sin^{-1} u(x)$	$\frac{1}{\sqrt{1-u^2}} \cdot u`(x)$
$\cos^{-1} u(x)$	$\frac{-1}{\sqrt{1-u^2}} \cdot u`(x)$
$\tan^{-1} u(x)$	$\frac{1}{\sqrt{1+u^2}} \cdot u`(x)$
$\sinh^{-1} u(x)$	$\frac{1}{\sqrt{1+u^2}} \cdot u`(x)$
$\cosh^{-1} u(x)$	$\frac{1}{\sqrt{u^2-1}} \cdot u`(x)$
$\tanh^{-1} u(x)$	$\frac{1}{\sqrt{1-u^2}} \cdot u`(x)$

⇒ $\sin^{-1} x \neq (\sin x)^{-1}$

$\sin^{-2} x = (\sin x)^{-2} = \frac{1}{\sin^2 x}$

$(\sin^{-2} x)^3 = \sin^{-6} x$

$(\sin^{-1} x)^3 \neq \sin^{-3} x$

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⇒ Find y'

(1) $y = \sin^{-1} 3x + \sin^{-2} 3x$

$$y' = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 + -2 \cdot (\sin 3x)^{-3} \cdot \cos 3x \cdot 3$$

(2) $y = (\sinh^{-1} 2x)^3 + \tanh^{-3} x$

$$y' = 3 (\sinh^{-1} 2x)^2 \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 +$$
$$-3 (\tanh x)^{-4} \cdot \operatorname{sech}^2 x$$

⇒ show that $\sinh^{-1} x = \ln (x + \sqrt{1+x^2})$ and it is odd Function

Proof

$y = \sinh^{-1} x$

$x = \sinh y = \frac{e^y - e^{-y}}{2}$

$2x = e^y - e^{-y}$ $* \, e^y$

$2x \cdot e^y = e^{2y} - 1$

$e^{2y} - 2x \cdot e^y - 1 = 0$

$(e^y)^2 - 2x(e^y) - 1 = 0$

$Ax^2 + Bx + c = 0$

$x = e^y$

$A = 1$, $B = -2x$, $c = -1$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore e^y = \frac{2x \pm \sqrt{4x^2 - 4(1)(-1)}}{2(1)} = \frac{2x \pm \sqrt{4x^2+4}}{2} = \frac{2x \pm 2\sqrt{x^2+1}}{2}$$

$$= x \pm \sqrt{x^2+1}$$

$$\therefore e^y = x + \sqrt{x^2+1}$$

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$\therefore\ y = \ln\left(x + \sqrt{1 + x^2}\right)$

$\because\ y = \sinh^{-1} x$

$\therefore\ \sinh^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right)$

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$$\sinh^{-1}(-x) = \ln\left(-x + \sqrt{1 + (-x)^2}\right)$$

$$\sinh^{-1}(-x) = \ln\left(-x + \sqrt{1 + x^2}\right)$$

$$\sinh^{-1}(-x) = \ln\left(-x + \sqrt{1 + x^2} \cdot \frac{-x - \sqrt{1 + x^2}}{-x - \sqrt{1 + x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{\cancel{x^2} - (\cancel{x^2} + 1)}{-x - \sqrt{1 + x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{-1}{-x - \sqrt{1 + x^2}}\right)$$

$$\sinh^{-1}(-x) = \ln\left(\frac{1}{x + \sqrt{1 + x^2}}\right)$$

$$\sinh^{-1}(-x) = \underbrace{\cancel{\ln 1}}_{=0} - \ln\left(x + \sqrt{1 + x^2}\right)$$

$$\sinh^{-1}(-x) = -\ln\left(x + \sqrt{1 + x^2}\right)$$

$\therefore\ \sinh^{-1}(x) = \ln\left(x + \sqrt{1 + x^2}\right)$ is odd Function

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